MATH:3770
Discussion Worksheet 8
March 20, 2024

The goal of this discussion is to build familiarity with sequences and their properties.

Let (a_n) be a sequence in $\mathbb{R}$. The following is a list of descriptions which may be applied to (a_n):

- is a sequence;
- is bounded;
- is monotone;
- converges to $a \in \mathbb{R}$;
- is bounded and monotone;
- diverges;
- diverges to $+\infty$;
- is Cauchy;
- is unbounded;
- is a subsequence of $\left(\frac{1}{n}\right)$.

Your goal is to perform three tasks:

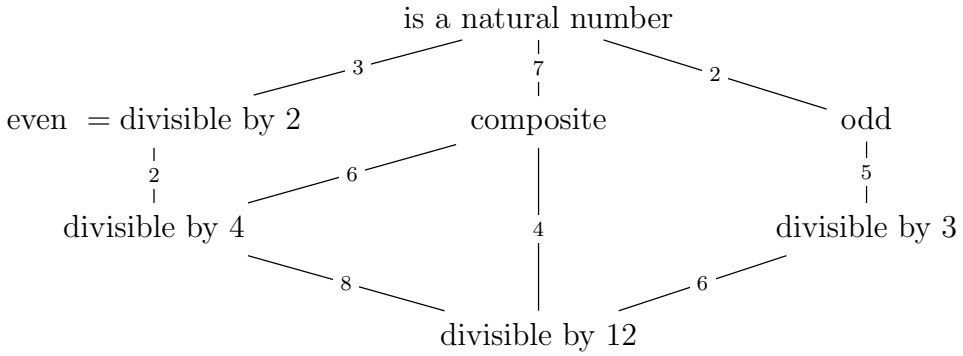
1. Give a precise definition of each term.
2. Nest the terms - that is, indicate which terms imply which other terms.
3. Provided an example which witnesses the nesting. For instance, if term A nests in term B, then you should provide an explicit example of a sequence (a_n) which satisfies B but not A.

[An example of (2) and (3) is provided on the back.]

Let n be a natural number. The following is a list of descriptions which may be applied to n:

- is a natural number;
- is even;
- is odd;
- is divisible by 2;
- is divisible by 4;
- is divisible by 3;
- is divisible by 12;
- is composite.

In the following diagram, we nest the terms and provide witness examples.



For instance, there is an edge between "composite" and "divisible by 12" which is labeled by 4. This indicates that all multiples of 12 are composite but not all composites are divisible by 12 (for instance, 4).