Teaching Statement

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"Why are numbers beautiful? It's like asking why is Beethoven's Ninth Symphony beautiful... I know numbers are beautiful. If they aren't beautiful, nothing is." (Paul Erdös)

Simply put, I both do and teach mathematics because I find it beautiful, and I enjoy sharing its beauty with others. One of my goals as an educator is to share this beauty with my students as I encourage them to see beyond rote computations and develop a mathematical mindset.

"These mundane 'useful' aspects would follow naturally as an almost trivial byproduct." (Paul Lockhart)

When I teach mathematics, my goal is not to convey to my students a set of procedures used to solve a relatively fixed set of problems. Rather, I want to teach them to think like a mathematician, to understand how solutions to problems came about, why they work, and how to generalize them. Thus, the goal is not to give them a toolbox, but to teach them to make their own tools. Ultimately, I view this as far more valuable than any set of memorized techniques.

Take as an example the use of the mnemonic FOIL to remember how to expand the product of binomials. It works; it's fast. This is to the good. However, I would rather teach my students to expand binomial products just by using the distributive law of multiplication. This is a little slower, both to teach and to perform. On the other hand, it allows for the expansion of products of non-binomials which FOIL does not. This method is also more robust. If you forget the acronym or what each letter means, FOIL becomes useless. Expansion by distributivity is much easier to retain. All you really need to remember is the statement of the distributive law, something probably introduced in a much earlier class, and that expanding products is just an application. Mucking around with pencil and paper will then inevitably result in correct expansion.

The emphasis I place on understanding conceptual building blocks reaches beyond simple algebraic tricks like FOIL. It underpins my entire philosophy of teaching, illustrating my belief that mathematical ideas are much easier to understand, use, and retain when they are firmly understood on the level of ideas. That said, I recognize that my own preference for the theoretical side of mathematics has a strong influence here. Many of my students will have different learning goals, often just wanting to know the technical tools they need for their discipline. Thus, while I emphasize the importance and utility of understanding underpinning concepts, I still make sure that my students see the practical techniques that will benefit them in their own areas of study.

To be clear, I am not opposed to teaching mnemonics like FOIL at all. As above, it is a fast way to expand the product of binomials. My point is that I believe it should be taught that such expansions can be done by hand and that such a computation is very natural. Once the conceptual building block is in hand, then I am entirely supportive of introducing computational tricks like FOIL.

"Mathematics, you see, is not a spectator sport." (George Pólya)

One rarely fully understands a mathematical idea without having used it. In accordance with this general wisdom, I always try to incorporate a component of participation when I teach. When leading a Calculus II discussion section for instance, I would would spend merely the first fifteen minutes in lecture style review (more if prompted by my students with questions) before breaking the class into small groups to work on problems. These were designed to range in difficulty from (a) near copies of worked examples from mini-lecture to (b) problems similar to those but with a higher level of computational complexity or requiring the use of techniques from previous lessons to (c) questions requiring some novel insight. An example of a type-c problem during first exposure to integration by parts was a "cyclic" integral of the sort $\int e^x \sin x \, dx$ before the trick to these had been formally presented.

While the students worked on these problems, I would circulate through the room answering questions and checking answers. Often, students would struggle with type-c questions. In part, this was by design. Struggling to answer a hard problem can be a good learning experience. Moreover, it has been my own experience as a student that I appreciate and remember a technique for solving a problem better if I have struggled a bit with the problem first. Still, the goal was not to overly frustrate my students or to cause distress if they could not find a solution. I made sure to come prepared with initial guiding hints and nudges for these problems. I was often pleasantly surprised by techniques they sometimes discovered to resolve these challenging questions.

My discussions with these small groups as I circulated also provided me with an opportunity for assessment, one quite different from grading homework or exams. I got to see my students problem solving processes in real time, ask questions, and give immediate feedback. At the end of discussion, when we gathered again as one group to discuss the problems, I could use those assessments to guide discussion.

I will end with a glimpse into Bertrand Russell's mathematical education and hope that none of my students ever need express a similar sentiment.

"I was made to learn by heart 'the square of the sum of two numbers is equal to the sum of their squares increased by twice their product.' I had not the vaguest idea what this meant, and when I could not remember the words, my tutor threw the book at my head, which did

not stimulate my intellect in any way." (Bertrand Russel)