Functions and Linear Equations

Module 2, Worksheet 3 and Module 3, Worksheet 1

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Announcements

- ALEKS Module 3 due Monday Sept. 15 at 11:59 pm
- Quiz Module 3 on Monday in class
- Metacognition Journal 2 due Sept. 15 (answer drawing on the study log)

- Office Hours: Thursday (1:00 - 2:00) and Friday (12:30 - 1:30)
- MathLab Hours: Tuesday 11:30 - 12:30

Quiz Post Mortem

- **Stats:** TBD
- Remember that the lowest 3 scores are dropped
- Make-up Quizzes: after you miss 3
- If a grading mistake was made, please tell me

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- $x \leq 3$
- $0 \leq x < 3$
- $x < 0$ or $x \geq 3$

Interval Notation

- $(3, \infty)$
- $(-\infty, 3]$
- $[0, 3)$
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Dividing Rational Functions

$$\frac{x^2 + x}{x - 5} \div \frac{x^3}{x^2 - 25}$$

3 / 25

Functions - Basics

A function is an input-output machine

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Example: The function which squares numbers

- Input: any number (*e.g.* 0, -2, 5, 20/7)
- Output: whatever the input is but squared (*e.g.* 0, 4, 25, 400/49)

Function Notation I

Expressing functions with words (as in the squaring function) is inefficient

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- Specify what types of inputs are allowed (*e.g.* whole numbers, positive numbers)
- Specify the output by writing
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- Consider the function f
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- Consider the function f
- Given by $f(x) = x^2$
- For x any number

Function Notation II

Consider the function f given by $f(x) = x^2$ for x any number.

- Inputs: any number
- Output: whatever the square of the input is

The output a function associates to an input is called the **value of the function at the input**. Finding this value is called **evaluation**.

- The value of the squaring function at 5 is 25
- If I were asked to evaluate the squaring function at 3 I would square 3. The answer is then 9.

When using function notation, say for a function h, the value of function h at input a is written $h(a)$.

- The value of the squaring function at 5 is $f(5) = 5^2 = 25$.

Function Notation III

If given a function $f(x) = \{$ some mathematical formula $\}$ and you are asked to

- finder the value of f at some input (*e.g.* 1), or
- evaluate f at some input (*e.g.* 1), or
- calculate $f(1)$

You do the following

1. Find all instances of x in the formula for the function.
2. Replace the x's with the input.
3. Simplify (if desired).

Example: Consider $f(x) = \frac{x^2-7}{x+1}$ for $x \neq -1$. To calculate $f(1)$ we

1. Find all instances of x in the formula for the function $f(x) = \frac{x^2-7}{x+1}$.
2. Replace the x's with the input $f(1) = \frac{1^2-7}{1+1}$
3. Simplify $f(1) = \frac{1^2-7}{1+1} = \frac{-6}{3} = -2$

Practice

- Let $f(x) = (x-1)^2$ for $x > 1$. Individually calculate $f(7)$. Compare with group members.

- Let $f(x) = 3x - 7$ for x a whole number. One of the following statements is objectionable

$$f(1) = -4 \text{ and } f(1/3) = -6$$

Which is it and why? Discuss in your groups.

Function Notation IV

You can evaluate functions at variable too.

- If f is a function then so is $f(x-3)$ or $f(x^2)$ or $f(2x)$

Example Let $f(x) = x^2 + 7$.

- $f(x-3)$ is that function which inputs x and outputs

$$\begin{aligned} f(x-3) &= (x-3)^2 + 7 \\ &= x^2 - 6x + 9 - 7 \\ &= x^2 - 6x + 2 \end{aligned}$$

Whenever you have $f(x) = \{$ some mathematical formula $\}$ and are asked to think about $f($some stuff$)$ you

1. Find all instances of x in the formula for the function
2. Replace the x's with some stuff

Function Notation V - Abuse of Notation

It is very common to see something like

$$\text{Consider the function } f(x) = \sqrt{x} \cdot \frac{1}{x}$$

without input values of x specified.

When this happens, you can assume that x is allowed to be any number which does not result in illegal operations (square roots of negatives, division by 0,...).

For $f(x) = \sqrt{x} \cdot \frac{1}{x}$ we allow x to be any non-negative number with $x \neq 0$.

Linear Functions, Linear Expressions

- Functions are usually given as $f(x) = \{$ some mathematical formula$\}$
- We classify such functions by features of the formula

When a function f is given by

$$\begin{aligned} f(x) &= ax + b && (a, b \text{ numbers}) \\ &= (\text{some number})x + (\text{some other number}) \end{aligned}$$

we say f is a **linear function**. The actual formula

$$ax + b$$

is called a **linear expression**

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Examples

- $f(x) = 7x + 2$ (linear function)
- $h(x) = \frac{9}{2}x + \pi$ (linear function)
- $-2x + 9.3$ (linear expression)
- $2x$ (linear expression)

An Aside (Definitions are Slippery Beasts)

- We're saying a linear expression is anything of the form $ax + b$ where a, b are numbers.
- Any equivalent expression is also called linear.
  - $3 + x$ is linear because it is the same as $x + 3$
  - $2(x + 7)$ is linear because it is the same as $2x + 14$

Solving Linear Equations I

A **linear equation** is

$$\{\text{some linear expression}\} = \{\text{some other linear expression}\}$$

when both expressions use the same variable (*e.g.* x).

Examples

$$3x + 1 = 6x - 8$$
$$x = x + 21$$
$$3x = 3x$$

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When you are asked **solve** a linear equation you are being asked

- "What if any numbers which when plugged in for x make the equality true?"

Such a value is called a **solution**.

Solving Linear Equations II

$$3x + 1 = 6x - 8 \quad (1)$$
$$x = x + 21 \quad (2)$$
$$3x = 3x \quad (3)$$

When you are asked **solve** a linear equation you are being asked

- "What if any numbers which when plugged in for x make the equality true?"

Such a value is called a **solution**.

- $x = 3$ is a solution to (1) because $3 \cdot 3 + 1 = 10 = 6 \cdot 3 - 8$
- $x = 3$ is **not** a solution to (2) because $3 \neq 3 + 21$
- $x = 3$ is a solution to (3) because $3 \cdot 3 = 3 \cdot 3$

Solving Linear Equations III

$$3x + 1 = 6x - 8 \quad (1)$$

$$x = x + 21 \quad (2)$$

$$3x = 3x \quad (3)$$

A linear equation can have

- exactly 1 solution (*e.g.* equation (1))
- no solutions at all (*e.g.* equation (2))
- infinitely many solutions (*e.g.* equation (3))

15 / 25

Solving Linear Equations IV, The Practice

To solve a linear equation do the following you attempt to **isolate the variable**. To do so

- Perform a valid mathematical operation to both sides the equals sign.
- Be sure to do the same operation on both sides.
- Repeat wit the goal of obtaining $x = \{\text{some number}\}$.

One of three things will happen

- You get $x = \{\text{some number}\}$. That number is the one and only solution.
- You get a false statement (*e.g.* 0=1). Then there is no solution at all.
- You get a statement which is always true (e.g. 0=0). Then any x is a solution.

Solving Linear Equations V, Workbook Example 4

Let's solve $3 - 2x = 7 - 3x$.

Step I: Eliminate terms without x from one side.

$$\begin{aligned} (3 - 2x) - 3 &= (7 - 3x) - 3 && \text{(subtract 3 from both sides)} \\ -2x &= 4 - 3x \end{aligned}$$

Step II: Eliminate terms with x from the other side.

$$\begin{aligned} (-2x) + 3x &= (4 - 3x) + 3x && \text{(add } 3x \text{ to both sides)} \\ x &= 4 \end{aligned}$$

We are done! There is exactly one solution: $x = 4$.

17 / 25

Solving Linear Equations V, Workbook Example 3

Let's solve $3x + 1 = -\frac{1}{2}(6 - 4x)$.

Step 0: I don't like fractions. Let's get rid of it.

$$2 \cdot (3x + 1) = 2 \cdot \big(-\frac{1}{2}\big)(6 - 4x) \qquad \text{(multiply both sides by 2)}$$
$$6x + 2 = 4x - 6$$

Step I: Eliminate terms without x from one side.

$$(6x + 2) - 2 = (4x - 6) - 2 \qquad \text{(subtract 2 from both sides)}$$
$$6x = 4x - 8$$

Step II: Eliminate terms with x from the other side.

$$(6x) - 4x = (4x - 8) - 4x \qquad \text{(subtract } 4x \text{ from both sides)}$$
$$2x = -8$$

Solving Linear Equations V, Workbook Example 3

Let's solve $3x + 1 = -\frac{1}{2}(6 - 4x)$.

After Step II: $2x = -8$

Step III: Remove coefficients from term with x.

$$(-x) \cdot \frac{1}{2} = (-8) \cdot \frac{1}{2} \qquad \text{(multiply both sides by } 1/2\text{)}$$
$$x = 4$$

We are done! There is exactly one solution: $x = -4$.

Practice

Problem Solve $2x - 2 = 3 - x$.

Instructions In your groups, work to solve the problem. Nominate a member or members to present a solution at the board. Which group presents will be chosen at random.

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Solving Linear Equations VI, A Generic Solution

Consider a linear equation

$$ax + b = cx + d$$

- If $a = c$ and $b \neq d$, then there is no solution.
- If $a = c$ and $b = d$, then any x is a solution.
- If $a \neq c$, then the one and only solution is

$$x = \frac{d - b}{a - c}.$$

The Basic Idea

We are often in the position of having to use math to solve a problem presented to us in non-mathematical symbols and/or coming from the real world.

- The sum of 3 times a number and 5 equals 7. What is the number?
- A book and a table together cost $300. The table costs $50 more than the book. What is the cost of the book.
- You earn $14 an hour. How long must you work to earn $400?

It is a useful skill to translate the real world problem into a mathematical problem. Then solve the math problem.

The Basic Procedure I

Let's assume we're working problems asking us to identify some unknown quantity or quantities.

Procedure

1. Identify the unknown quantity.
2. Give it a name.
3. Right down any facts as mathematical equation(s).
4. Solve the resulting equation(s).
5. Ask if the solution makes sense in the context of the problem.

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Example: The sum of 3 times a number and 5 equals 7. What is the number?

1. We want to find an unknown number.
2. Call the number x.
3. $3x + 5 = 7$.
4. $x = 2/3$.
5. There are no real world issues with this answer.

The Basic Procedure II

Problem: You earn \$14 an hour. How long must you work to earn \$400?

1. I want to know how long (in hours) to work.
2. Let's call that times (in hours) T.
3. I get payed \$14 for each our I work. So, if I work for t hours I get $14t$ dollars. If I get payed \$400 after T hours, then $14T = 400$.
4. When I solve $14T = 400$ I obtain $T = 200/7 \approx 28.57$.
5. I cannot reasonable be expected to be payed for fractional time. So I need to complete the hour to be payed. So the answer is 29 hours.

Practice

Instructions In your groups, work to solve the following problem. Nominate a member or members to present a solution at the board. Which group presents will be chosen at random.

Problem The perimeter of a square is 64in. What is its area?