Properties of Functions

Module 5, Worksheet 2

Announcements

- ALEKS Module due **Monday** Sept. 29 at 11:59 pm
- Quiz Module 5 **Monday** Sept. 29

- First Midterm **Oct. 2**. See ICON page for details and review sheet.
- First ALEKS PIE check (for 45% pie completion) due **Oct. 3**

- Office Hours: Thursday (1:00 - 2:00) and Friday (12:30 - 1:30) in B20G MLH
- MathLab Hours: Tuesday 11:30 - 12:30

- Bingo!

- Today: second worksheet in Module 5.

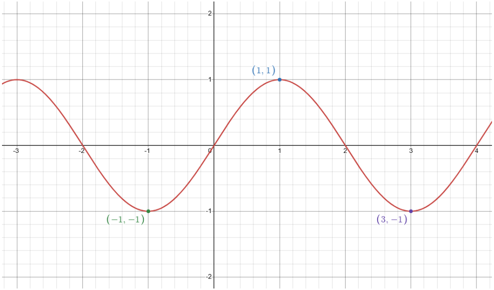
Functions - Basics

The main idea to be explored today is that functions have properties which can be detected either by

- inspecting the formula $y = f(x)$ and doing work, or
- Visually inspecting the graph.

The second option is sometimes easier.

Example The following is a graph of $y = \sin(\pi x/2)$:



There are peaks and troughs to this graph, *e.g.* at $(1, 1)$, $(-1, -1)$, and $(3, -1)$.

Visually this is obvious, but extracting this from $y = \sin(\pi x/2)$ is a typical Calc I problem.

Increasing and Decreasing Functions I

A function $y = f(x)$ is **strictly increasing** when $f(x_1) < f(x_2)$ whenever $x_1 < x_2$

- *e.g.* it should be that $f(1) < f(2)$ because $1 < 2$.

Graphically, this manifests as the graph going up as you move right.

A function $y = f(x)$ is **strictly decreasing** when $f(x_1) > f(x_2)$ whenever $x_1 < x_2$

- *e.g.* it should be that $f(1) > f(2)$ because $1 < 2$.

Graphically, this manifests as the graph going down as you move right.

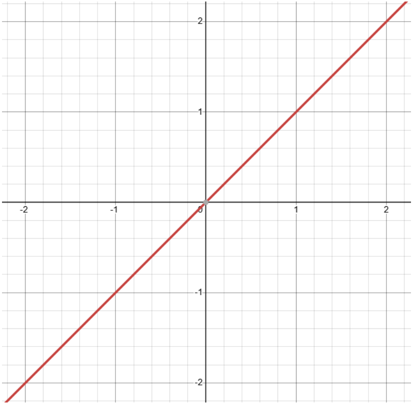
A function $y = f(x)$ is **constant** when $f(x_1) = f(x_2)$ for all inputs x_1, x_2.

- *e.g.* it should be that $f(1) = f(2)$.

Graphically, this manifests as the graph being flat.

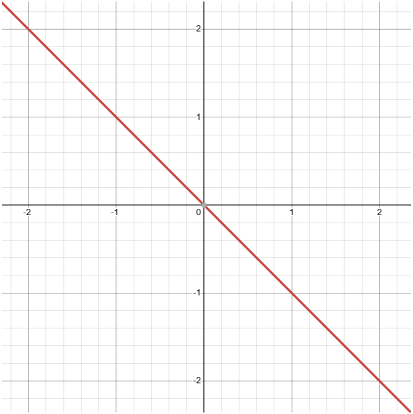
Increasing and Decreasing Functions II

$y = x$ is increasing.



A linear function is increasing when it has positive slope.

$y = -x$ is increasing

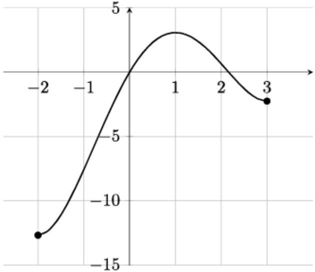


A linear function is decreasing when it has negative slope

Increasing and Decreasing Functions III

A function can be increasing, decreasing or constant on an interval.

(a)

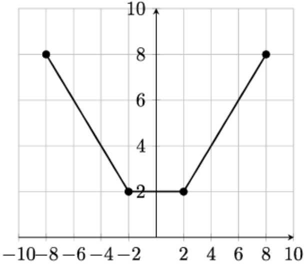


(strictly) increasing: ____________________

(strictly) decreasing: ____________________

constant: ____________________

(b)



(strictly) increasing: ____________________

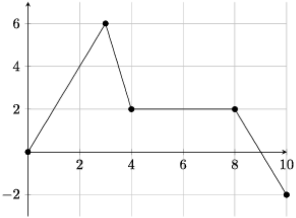
(strictly) decreasing: ____________________

constant: ____________________

Your Turn

Exercise 3. Write the intervals for which the functions graphed below are increasing or decreasing.

(a)

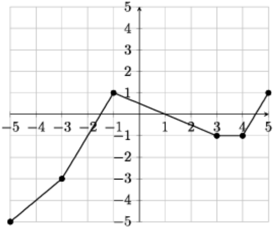


(strictly) increasing: ____________

(strictly) decreasing: ____________

constant: ____________

(b)



(strictly) increasing: ____________

(strictly) decreasing: ____________

constant: ____________

Global Extrma I

A function f has a **global maximum at** a when $f(a)$ is greater than or equal to all values $f(x)$. That is, $f(a)$ is the largest value f takes on.

Important Terminology Suppose that f has a global maximum at a.

- The **value** of the maximum if $f(a)$.
- $f(a)$ is **the maximum**.

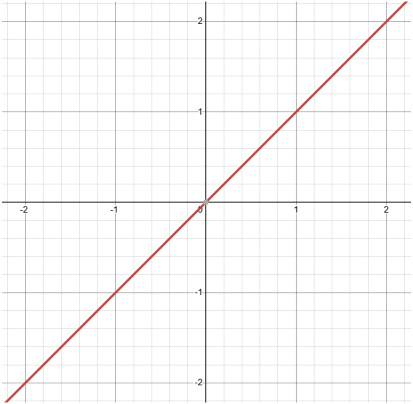
Graphically: No points on the graph has y value greater than $f(a)$. The entire graph is below $y = f(a)$.

A function f has a **global minimum at** a when $f(a)$ is less than or equal to all values $f(x)$. That is, $f(a)$ is the smallest value f takes on.
Graphically: No points on the graph has y value less than $f(a)$. The entire graph is above $y = f(a)$.

- A value which is either a global maximum or a global minimum is called an **global extremum** or **global extreme value**.

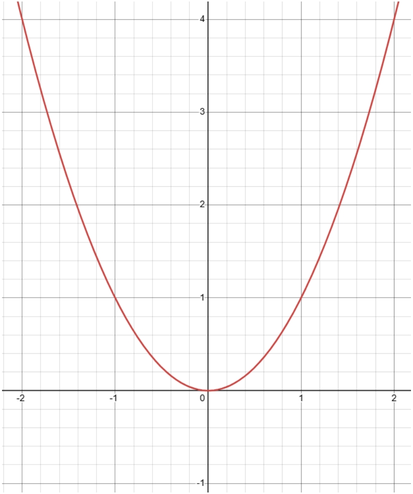
Global Extrema II

$y = x$ is has no extreme values.



A linear function has extreme values only when slope is 0

$y = x^2$ has global minimum of 0 at 0



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Local Extrema

A function f has a **local maximum at** a when $f(a)$ is greater than or equal to all values $f(x)$ for x near a. That is, $f(a)$ is the largest value f takes on near a.

- *Graphically:* The graph of f has a peak or hilltop at $(a, f(a))$.

A function f has a **local minimum at** a when $f(a)$ is less than or equal to all values $f(x)$ for x near a. That is, $f(a)$ is the smallest value f takes on near a.

- *Graphically:* The graph of f has a trough or valley at $(a, f(a))$.

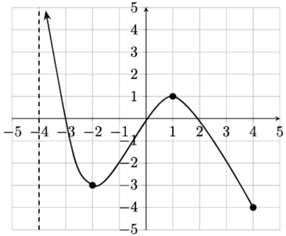
Terminology A value which is either a global maximum or a global minimum is called an **global extremum** or **global extreme value**.

Observations A function can have at most one global maximum. It may have many local maxima. A global maximum is a local maximum.

Example 7

Example 7. Identify any local maxima and minima along with the absolute maximum and minimum of each function on the interval shown. Write "none" if no such value exists.

(a)



domain: ____________________

range: ____________________

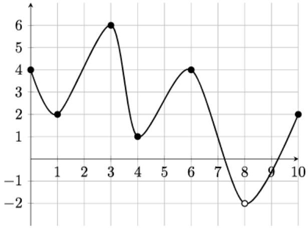
local maxima: ______________

local minima: ______________

absolute maximum: __________

absolute minimum: __________

(b)



domain: ____________________

range: ____________________

local maxima: ______________

local minima: ______________

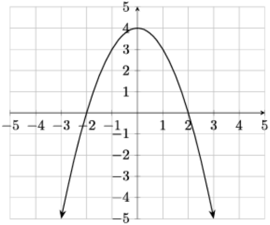
absolute maximum: __________

absolute minimum: __________

Your Turn

Exercise 8. Identify any local maxima and minima along with the absolute maximum and minimum of each function on the interval shown. Write "none" if no such value exists.

(a)



domain: ____________________

range: ____________________

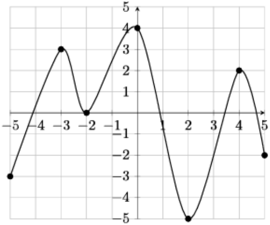
local maxima: ____________________

local minima: ____________________

absolute maximum: ____________

absolute minimum: ____________

(c)



domain: ____________________

range: ____________________

local maxima: ____________________

local minima: ____________________

absolute maximum: ____________

absolute minimum: ____________

Even and Odd Functions

A function f is **even** when for all x in its domain there holds $f(x) = f(-x)$.

- *Graphically:* The graph of f is symmetric about the y-axis.

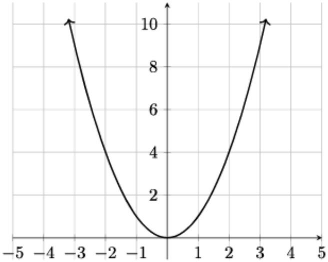
A function f is **odd** when for all x in its domain there holds $f(-x) = -f(x)$.

- *Graphically:* The graph of f is symmetric about the origin (*i.e.* does not change if rotated 180°).

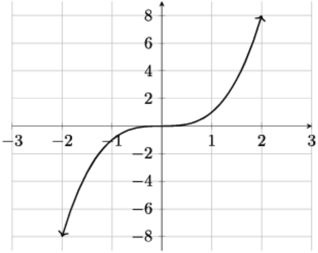
Notice The function $y = x^n$ is even as a function when n is an even number. The function $y = x^n$ is odd as a function when n is an odd number.

Example 15

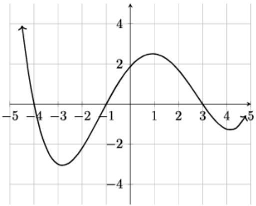
(a)



(b)



(c)

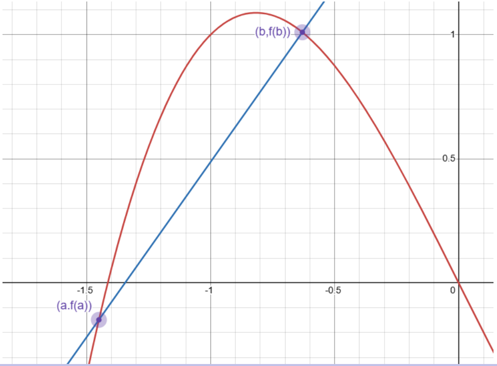


Average Rate of Change

If f is a function whose domain contains the interval $[a, b]$ for $a \neq b$, then the **average rate of change of** f **over the interval** $[a, b]$ is the value

$$\frac{f(b) - f(a)}{b - a}$$

That is, the slope of the line between $(a, f(a))$ and $(b, f(b))$.



Example 11a

Problem Find the average rate of change of $f(x) = x^2 + 4x - 2$ from $x = 0$ to $x = 3$.

Example 11a

Problem Find the average rate of change of $f(x) = x^2 + 4x - 2$ from $x = 0$ to $x = 3$.

Solution
Step 0 Recall the formula:

$$\text{average rate of change from } a \text{ to } b = \frac{f(b) - f(a)}{b - a}$$

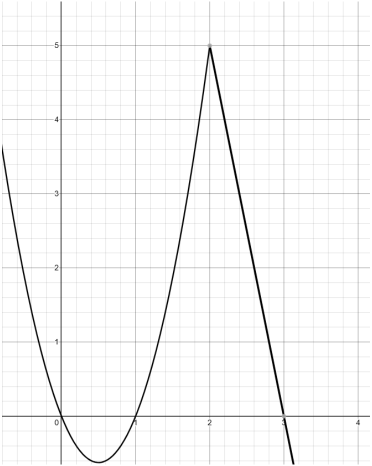
Step 1 Identify a, b. Set $a = 0$ and $b = 3$.

Step 2 Plug into formula:

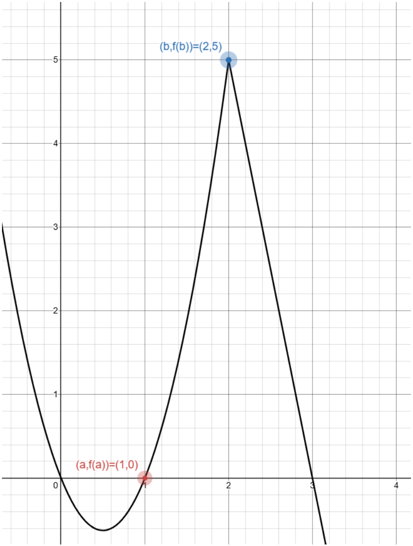
$$\begin{aligned} \text{average rate of change from 0 to 3} &= \frac{f(3) - f(0)}{3 - 0} \\ &= \frac{(3^2 + 4 \cdot 3 - 2) - (0^2 + 4 \cdot 0 - 2)}{3} \\ &= \frac{15}{3} = \boxed{5} \end{aligned}$$

Example 11b

Problem Find the average rate of change of the following graphed function f from $x = 1$ to $x = 2$



Example 11b Solution



1. Here $a = 1$ and $b = 2$
2. Find $f(a) = f(1) = 0$ and $f(b) = f(2) = 5$ using the graph
3. Plug into formula

$$\frac{f(b) - f(a)}{b - a} = \frac{5 - 0}{2 - 1}$$

4. So the average rate of change is 5.

Your Turn

Exercise 12(a) Find the average rate of change of $f(x) = x^3 + 4$ from $x = -1$ to $x = 3$.

Exercise 12(b) Find the average rate of change of the following graphed function from $x = -2$ to $x = 2$.